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Dynamics of D1-brane in I-brane background

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Abstract: This paper is devoted to the study of the effective field theory description of the probe D1-brane in the background of the system of two stacks of fivebranes in type IIB theory that intersect on $R^{1,1}$. We study the properties of the Dirac-Born-Infeld action for D1-brane moving in this background. We will argue that this action is invariant under an additional symmetry in the near horizon limit and that this new symmetry is closely related to the enhanced symmetry of the I-brane background considered recently in [1]. We also solve explicitly the equation of motion of D1-brane in the near horizon limit.

Keywords: Tachyon Condensation, D-branes.

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Contents

1. Introduction and summary

One of the most outstanding problems in string theory is the problem of string dynamics in the time dependent background. On the other hand there was great recent progress pioneered by work of A. Sen [2–5] that showed that the rolling tachyon could be used to describe an decay of an unstable D-brane. It is very nice that Dirac-Born-Infeld (DBI)-like tachyon effective action [6–9] can capture most of the aspects of the time dynamics of the tachyon.

The problem of the time dependence of the string theory was also analyzed from different point of view in the paper [11][1] where the time dependent motion of probe Dp-brane in the background of N NS5-branes was considered. Among many interesting results derived there was an observation that the radial motion of such a probe Dp-brane can be mapped to the time depend tachyon like condensation. It is then very tempting to conjecture that tachyon could have geometrical origin. Kutasov's paper is also an example how probe Dp-brane dynamics could be useful for the study of properties of given background. In fact the probe D-brane analysis was very successful in the study of matrix theory (See [33, 34] for reviews and extensive list of references.) and AdS/CFT correspondence (See for example [35].) For that reason it seems to be natural to analyze the dynamics of the probe D-brane in all interesting backgrounds. One such an example of a nontrivial string theory background was recently considered in a remarkable paper by N. Itzaki, D. Kutasov and N. Seiberg [1]. This background consists two stacks of fivebranes in type IIB theory that intersect on $R^{1,1}$. Very careful analysis presented there discovered many interesting results. For example, it was shown that there is an enhancement of the symmetry in the near horizon geometry of given background that in some sense mixes the world volume coordinate of I-brane with the transverse ones.[2] This property was then analyzed in [1] from the point

[1] Some related works are [12–32].

[2] This point was also confirmed in [36].

of view of the effective field theory living on the intersecting NS5-branes and also from the perspective of the closed string moving in this background. Unfortunately there is not enough place to review all results derived in this paper and we recommend the original paper [1] for more details with profound physical interpretation.

The goal of this paper is much modest. We will study the classical dynamics of the probe D1-brane in the I-brane background, where one can hope, according to arguments given above, that this dynamics could reflect some of the nontrivial predictions given in paper [1]. To do this we will carefully examine the conserved charges for D1-brane moving in given background. We will see that in the near horizon limit of the I-brane geometry one can find on the world volume of D1-brane a new transformation. It turns out that this-scaling like transformation-is symmetry of the action in case of the pure time dependent dynamics on the world volume of D1-brane and also in case when all world volume scalar modes are spatial dependent only.[3] Then with the help of the new conserved charge we will be able to solve the equations of motion explicitly. Following [12, 29] we could expect that this time evolution of the probe D1-brane in I-brane background has holographic description in a dual $2+1$ dimensional Little String Theory [1].

Despite of the fact that the analysis given in this paper cannot much to say to the holographic formulation of I-dynamics we can still hope that it could be helpful for the study of the interesting properties of I-brane background. For example, an emergence of the scaling symmetry in the near horizon region is related to the enhancement of the world volume symmetry of I-brane from $(1,1)$ to $(2,1)$. We also hope that the conformal field theory description of D-brane dynamics in I-brane background will be determined and the solutions found in this paper could arise as a classical limit of the exact boundary state analysis. On the other hand we still believe that the DBI analysis of the probe could be extended in several directions. First of all, we are going to study the spatial dependent solutions of the D1-brane in the I-brane background. We can also study the dynamics of D1-brane probe in the background where some of the NS5-branes are not coincident. Another interesting problem seems to be the analysis of probe Dp-brane dynamics in the background of $1+1$ dimensional intersection of two orthogonal stacks of NS5-branes in the type IIA theory or in the background of $1+1$ dimensional intersection of two orthogonal stack of D5-branes in type IIB theory.

The organization of the paper is as follows. In the next section 2 we will review the properties of the DBI action in general background. We will also study the symmetry properties of given action and we will outline the general prescription how to obtain charges that either are conserved or obey the equation where on the left hand side is a divergence of given current while the right hand side contains an anomaly term that is a result of the explicit breaking of given symmetry in given action. In section 3 we will introduce the lagrangian density for probe D1-brane in I-brane background and determine some conserved quantities that follow from the symmetry of D1-brane action in I-brane background. In section 4 we will solve the equation of motion for D1-brane in the near horizon limit where all modes on the world volume of D1-brane are homogeneous. We find explicit time

[3]The case of spatial dependent scalar fields will be analyzed in separate publication.

dependence of the radial modes that describe the radial position of D1-brane. We will also find the relation between the dilatation symmetry that emerges on the world volume of D1-brane and the new symmetry that was discovered in the paper [1].

2. DBI action

The starting point of our analysis is Dirac-Born-Infeld action for Dp-brane in a general background

$$S = -\tau_p \int d^{p+1}x e^{-\Phi} \sqrt{-\det \mathbf{A}_{\mu\nu}} \,, \qquad \mathbf{A}_{\mu\nu} = \gamma_{\mu\nu} + F_{\mu\nu} \,, \tag{2.1}$$

where τ_p is Dp-brane tension, $\Phi(X)$ is dilaton and where $\gamma_{\mu\nu}$, $\mu,\nu = 0,\ldots,p$ is embedding of the metric to the world volume of Dp-brane

$$\gamma_{\mu\nu} = g_{MN}\partial_\mu X^M \partial_\nu X^N \,, \qquad M,N = 0,\ldots,9 \,. \tag{2.2}$$

In (2.1) the form $F_{\mu\nu}$ is defined as

$$F_{\mu\nu} = b_{MN}\partial_\mu X^M \partial_\nu X^N + \partial_\mu A_\nu - \partial_\nu A_\mu \,, \tag{2.3}$$

where b_{MN} are components of NS two form. Thanks to the diffeomorphism invariance of the world volume theory it is natural to fix some space time coordinates X^M to be equal to the world volume coordinates x^μ:

$$X^\mu = x^\mu \,, \mu = 0,\ldots,p \,. \tag{2.4}$$

Note that this choice generally leads to the induced metric $\gamma_{\mu\nu}$ and two form $F_{\mu\nu}$ to be functions of x^μ and X^I where $I,J,\ldots = p+1,\ldots,D$ label coordinate transverse to the world volume of Dp-brane:

$$\begin{aligned}
\gamma_{\mu\nu} &= g_{\mu\nu}(x^\mu,X^I) + g_{\mu I}(x^\mu,X^I)\partial_\nu X^I + g_{J\nu}(x^\mu,X^I)\partial_\mu X^J + \\
&\quad + g_{IJ}(x^\mu,X^I)\partial_\mu X^I \partial_\nu X^J \,, \\
F_{\mu\nu} &= b_{\mu\nu}(x^\mu,X^I) + b_{\mu I}(x^\mu,X^I)\partial_\nu X^I + b_{J\nu}(x^\mu,X^I)\partial_\mu X^J + \\
&\quad + b_{IJ}(x^\mu,X^I)\partial_\mu X^I \partial_\nu X^J + \partial_\mu A_\nu - \partial_\mu A_\nu \,.
\end{aligned} \tag{2.5}$$

In the following we will consider the case when the background metric and two form field are block diagonal so that $g_{\mu I} = g_{J\nu} = b_{I\nu} = 0$. Now the equation of motion for $X^K, K = p+1,\ldots,9$ takes the form

$$\begin{aligned}
&\partial_K[e^{-\Phi}]\sqrt{-\det \mathbf{A}} + \frac{e^{-\Phi}}{2}\times \\
&\times \left[\partial_K g_{\mu\nu} + \partial_K g_{IJ}\partial_\mu X^I \partial_\nu X^J + +\partial_K b_{\mu\nu} + \partial_K b_{IJ}\partial_\mu X^I \partial_\nu X^J\right] \left(\mathbf{A}^{-1}\right)^{\nu\mu} \sqrt{-\det \mathbf{A}} - \\
&-\partial_\mu \left[e^{-\Phi} g_{KI}\partial_\nu X^I \left(\mathbf{A}^{-1}\right)^{\nu\mu}_S \sqrt{-\det \mathbf{A}}\right] - \\
&-\partial_\mu \left[e^{-\Phi} b_{KI}\partial_\nu X^I \left(\mathbf{A}^{-1}\right)^{\nu\mu}_A \sqrt{-\det \mathbf{A}}\right] = 0 \,,
\end{aligned} \tag{2.6}$$

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where

$$\left(\mathbf{A}^{-1}\right)_S^{\mu\nu} = \frac{1}{2}\left(\left(\mathbf{A}^{-1}\right)^{\nu\mu} + \left(\mathbf{A}^{-1}\right)^{\mu\nu}\right), \quad \left(\mathbf{A}^{-1}\right)_A^{\mu\nu} = \frac{1}{2}\left(\left(\mathbf{A}^{-1}\right)^{\nu\mu} - \left(\mathbf{A}^{-1}\right)^{\mu\nu}\right). \tag{2.7}$$

Finally, we should also determine the equation of motion for the gauge field A_μ:

$$\partial_\nu \left[e^{-\Phi}\left(\mathbf{A}^{-1}\right)_A^{\nu\mu}\sqrt{-\det \mathbf{A}}\right] = 0\,. \tag{2.8}$$

The power of the lagrangian formalism is that it is manifestly covariant. Then it is natural to determine all possible conserved charges or currents, whose divergences are nonzero and that correspond to some explicitly broken symmetry in given action. The knowledge of these currents then considerably simplifies the analysis of the dynamics of Dp-brane. To do this we will be more general and start with the action

$$S = \int d^D x \mathcal{L}(m^A, \phi^I(x), \partial_\mu \phi^I(x))\,, \tag{2.9}$$

where $m^A\,, A = 1, \dots, K$ are parameters that are contained in the action and where ϕ^I are dynamical fields. Let us now consider the variation of the action under general transformation of coordinates x^μ in the form

$$x'^\mu = x^\mu + \delta x^\mu\,, \tag{2.10}$$

where the infinitesimal δx^μ is specified by a set of infinitesimal parameters ω^i

$$\delta x^\mu = X_i^\mu(x)\delta\omega^i\,. \tag{2.11}$$

Under such a transformation the field ϕ^I will, in general, transforms. Thus

$$\phi'^I(x') = \phi^I(x) + \delta\phi^I(x)\,, \tag{2.12}$$

where $\delta\phi^I$ is also specified by parameters $\delta\omega^i$ so

$$\delta\phi^I = \Phi_i^I(x)\delta\omega^i\,, \tag{2.13}$$

where Φ^I are functionals of ϕ^J and functions of x. Evidently, the total variation $\delta\phi^I$ derives both from the variation of the field function and from the variation of its argument:

$$\phi'^I(x') = \phi'^I(x+\delta x) = \phi'^I(x) + \partial_\mu\phi^I(x)\delta x^\mu = \phi^I(x) + \delta_0\phi^I(x) + \partial_\mu\phi^I(x)\delta x^\mu\,. \tag{2.14}$$

Thus we find

$$\delta_0\phi^I(x) = \delta\phi^I(x) - \partial_\mu\phi^I(x)\delta x^\mu = \left[\Phi_i^I - \partial_\mu\phi^I\right]\delta\omega^i\,. \tag{2.15}$$

Let us also presume that the action is invariant under these symmetry transformations when parameters m^A transform as well

$$m'^A = m^A + \delta m^A = m^A + \Omega_i^A(\phi, x)\delta\omega^i\,. \tag{2.16}$$

Then the variation of the action takes the form

$$\delta S = \int \delta(d^D x)\mathcal{L} + \int d^D x \delta\mathcal{L}\,, \tag{2.17}$$

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where $\delta\mathcal{L}$ is the variation in the lagrangian density caused by above variation of x^μ, ϕ^I and m^A and $\delta(d^Dx)$ is the variation of the integration measure caused by variation of x^μ. In fact

$$\delta\left(d^Dx\right) = d^Dx' - d^Dx = \partial_\mu(X^\mu_i\omega^i)d^Dx\,. \tag{2.18}$$

Now remember that the variation of the lagrangian density is equal to

$$\delta\mathcal{L}(x) = \delta_0\mathcal{L}(x) + \frac{\delta\mathcal{L}}{\delta m^A}\delta m^A + \partial_\mu\mathcal{L}\delta x^\mu\,, \tag{2.19}$$

where the variation $\delta_0\mathcal{L}$ is caused by variation of $\delta_0\phi^I$. Then the variation of S is

$$\begin{aligned}\delta S = & \int d^Dx\partial_\mu\left(\mathcal{L}X^\mu_i\delta\omega^i + \frac{\delta\mathcal{L}}{\delta\partial_\mu\phi^I}\delta_0\phi^I\right) + \int d^Dx\frac{\delta\mathcal{L}}{\delta m^A}\Omega^A_i\delta\omega^i + \\ & + \int d^Dx\left[\frac{\delta\mathcal{L}}{\delta\phi^I} - \partial_\mu\frac{\delta\mathcal{L}}{\delta\partial_\mu\phi^I}\right]\delta_0\phi^I\,.\end{aligned} \tag{2.20}$$

If the fields obey the equation of motion that the expression on the last line vanishes. Then for the fields that obey the equation of motion the variation of the action is equal to

$$\delta S = -\int d^Dx\partial_\mu(j^\mu_i\delta\omega^i) + \int d^Dx\frac{\delta\mathcal{L}}{\delta m^A}\Omega^A_i\delta\omega^i\,, \tag{2.21}$$

where

$$j^\mu_i = \left(-\mathcal{L}\delta^\mu_\nu + \frac{\delta\mathcal{L}}{\delta\partial_\mu\phi^I}\partial_\nu\phi^I\right)X^\nu_i - \frac{\delta\mathcal{L}}{\delta\partial_\mu\phi^I}\Phi^I_i\,. \tag{2.22}$$

Now suppose that S is invariant when the variations are parameterized by constant $\delta\omega^i$. Then (2.21) implies

$$0 = \int d^Dx\left(-\partial_\mu j^\mu_i + \frac{\delta\mathcal{L}}{\delta m^A}\Omega^A_i\right)\delta\omega^i\,. \tag{2.23}$$

Consequently we get that the current j^μ_i obeys the equation

$$\partial_\mu j^\mu_i = \frac{\delta\mathcal{L}}{\delta m^A}\Omega^A_i\,. \tag{2.24}$$

In case when Ω^A_i vanish we obtain familiar result that the charge

$$Q_i(t) \equiv \int d^{D-1}xj^0_i \tag{2.25}$$

is constant, independent on t.

Now we apply this general discussion to the case of Dp-brane where as we know, the world volume fields are X^I and A_μ. Considering parameters m^A, we will introduce them into the action in case when the action is not invariant without their transformations as well.

As the first example we determine components of the world volume stress energy tensor. To do this we consider following transformation

$$x'^\mu = x^\mu + \epsilon^\mu\,, \tag{2.26}$$

where $\epsilon^\mu = const$. This transformation corresponds to

$$X^\mu_\nu = \delta^\mu_\nu \, . \tag{2.27}$$

Under this transformation the world volume fields transform as follows

$$\begin{aligned} X'^I(x') &= X^I(x) \Rightarrow \Phi^{X^I}_\mu = 0 \, , \\ A'_\mu(x') &= A_\nu(x) \left(\frac{\partial x'^\mu}{\partial x^\nu}\right)^{-1} = A_\nu(x)\delta^\nu_\mu = A_\mu(x) \Rightarrow \Phi^{A_\mu}_\nu = 0 \, . \end{aligned} \tag{2.28}$$

Since the action is manifestly invariant under these transformations there is no need to presume that the parameters m^A transform as well and the right hand side of the equation (2.24) is zero. Now using (2.27) and (2.28) we obtain the components of the world volume stress energy tensor

$$\begin{aligned} j^\mu_\nu \equiv T^\mu_\nu &= \left(-\mathcal{L}\delta^\mu_\kappa + \frac{\delta \mathcal{L}}{\delta \partial_\mu X^I}\partial_\kappa X^I + \frac{\delta\mathcal{L}}{\delta\partial_\mu A_\sigma}\partial_\kappa A_\sigma\right)\delta^\kappa_\nu \\ &= \tau_p e^{-\Phi}\left(\delta^\mu_\nu - g_{IJ}\partial_\kappa X^J \left(\mathbf{A}^{-1}\right)^{\kappa\mu}_S \partial_\nu X^I - b_{IJ}\partial_\kappa X^J \left(\mathbf{A}^{-1}\right)^{\kappa\mu}_A \partial_\nu X^I - \partial_\nu A_\sigma \left(\mathbf{A}^{-1}\right)^{\sigma\mu}_A\right) \times \\ &\quad \times \sqrt{-\det \mathbf{A}} \end{aligned} \tag{2.29}$$

that obey the equation

$$\partial_\mu T^\mu_\nu = 0 \, . \tag{2.30}$$

Even if it seems that the analysis performed above is well known we mean that it was useful to do it. Especially the case when the symmetry is explicitly broken will be important in the next section.

3. D1-brane in the background of I-brane

In this section we will study the dynamics of D1-brane in the background studied in the work [1]. Namely, we consider the intersection of two stack of NS5-branes. We have k_1 NS5-branes extended in $(0, 1, 2, 3, 4, 5)$ directions and the set of k_2 NS5-branes extended in $(0, 1, 6, 7, 8, 9)$ directions. Let us define

$$\begin{aligned} \mathbf{y} &= (x^2, x^3, x^4, x^5) \, , \\ \mathbf{z} &= (x^6, x^7, x^8, x^9) \, . \end{aligned} \tag{3.1}$$

We have k_1 NS5-branes localized at the points $\mathbf{z}_n$ $n = 1, \dots, k_1$ and k_2 NS5-branes localized at the points $\mathbf{y}_a \, , a = 1 \dots, k_2$. Every pairs of fivebranes from different sets intersect at different point $(\mathbf{y}_a, \mathbf{z}_n)$. The supergravity background corresponding to this configuration takes the form

$$\begin{aligned} \Phi(\mathbf{z}, \mathbf{y}) &= \Phi_1(\mathbf{z}) + \Phi_2(\mathbf{y}) \, , \\ g_{\mu\nu} &= \eta_{\mu\nu} \, , \mu, \nu = 0, 1 \, , \\ g_{\alpha\beta} &= e^{2(\Phi_2 - \Phi_2(\infty))}\delta_{\alpha\beta} \, , \mathcal{H}_{\alpha\beta\gamma} = -\epsilon_{\alpha\beta\gamma\delta}\partial^\delta\Phi_2 \, , \alpha, \beta, \gamma, \delta = 2, 3, 4, 5 \, , \\ g_{pq} &= e^{2(\Phi_1 - \Phi_1(\infty))}\delta_{pq} \, , \mathcal{H}_{pqr} = -\epsilon_{pqrs}\partial^s\Phi_1 \, , p, q, r, s = 6, 7, 8, 9 \, , \end{aligned} \tag{3.2}$$

where Φ on the first line means the dilaton and where

$$
\begin{aligned}
e^{2(\Phi_1-\Phi_1(\infty))} &= 1+\sum_{n=1}^{k_1}\frac{l_s^2}{|\mathbf{z}-\mathbf{z}_n|^2}\,,\\
e^{2(\Phi_2-\Phi_2(\infty))} &= 1+\sum_{a=1}^{k_2}\frac{l_s^2}{|\mathbf{y}-\mathbf{y}_a|^2}\,.
\end{aligned}
\tag{3.3}
$$

Our goal is to study properties of this background from the point of view of D1-brane probe when $\mathbf{z}_n=\mathbf{y}_a=0$. The action for D1-brane moving in given background takes the form

$$
S=-\tau_1\int d^2x e^{-\Phi}\sqrt{-\det\mathbf{A}}\,,
\tag{3.4}
$$

where we have implicitly used the static gauge so that the matrix $\mathbf{A}$ is equal to

$$
\mathbf{A}_{\mu\nu}=g_{\mu\nu}+g_{IJ}\partial_\mu X^I\partial_\nu X^J+b_{IJ}\partial_\mu X^I\partial_\nu X^J+\partial_\mu A_\nu-\partial_\nu A_\mu\,,\qquad I,J=2,\dots,9\,.
\tag{3.5}
$$

To simplify notation let us denote

$$
e^{2(\Phi_1-\Phi_1(\infty))}=H_1(\mathbf{z})\,,\qquad e^{2(\Phi_2-\Phi_2(\infty))}=H_2(\mathbf{y})\,,
\tag{3.6}
$$

where for coincident branes we have

$$
H_1=1+\frac{k_1l_s^2}{|\mathbf{z}|^2}\,,\qquad H_2=1+\frac{k_2l_s^2}{|\mathbf{y}|^2}\,.
\tag{3.7}
$$

Let us now consider the probe in the near horizon limit where

$$
\frac{k_1l_s^2}{|\mathbf{z}|^2}\gg1\,,\qquad \frac{k_2l_s^2}{|\mathbf{y}|^2}\gg1
\tag{3.8}
$$

so that we can write

$$
H_1=\frac{\lambda_1}{|\mathbf{z}|^2}\,,\qquad \lambda_1=k_1l_s^2\,,\qquad H_2=\frac{\lambda_2}{|\mathbf{y}|^2}\,,\qquad \lambda_2=k_2l_s^2\,.
\tag{3.9}
$$

In the near horizon limit the action takes the form

$$
S=-\tau_1\int d^2x\frac{1}{g_1g_2\sqrt{H_1H_2}}\sqrt{-\det\mathbf{A}}\,,
\tag{3.10}
$$

where

$$
\begin{aligned}
\mathbf{A}_{\mu\nu} &= \eta_{\mu\nu}+H_1\delta_{pr}\partial_\mu Z^p\partial_\nu Z^r+H_2\delta_{\alpha\beta}\partial_\mu Y^\alpha\partial_\nu Y^\beta+\\
&\quad +B_{pr}\partial_\mu Z^p\partial_\nu Z^r+B_{\alpha\beta}\partial_\mu Y^\alpha\partial_\nu Y^\beta+\partial_\mu A_\nu-\partial_\nu A_\mu\,,
\end{aligned}
\tag{3.11}
$$

and where $g_1=e^{\Phi_1(\infty)}, g_2=e^{\Phi_2(\infty)}$. According to arguments given in the paper [1] we can expect that D1-brane that moves in the near horizon limit possess some additional symmetry. In fact, it is easy to guess the form of such a transformation

$$
Z'^p(x')=\Gamma Z^p(x)\,,\qquad Y'^\alpha(x')=\Gamma^{-1}Y^\alpha(x)\,,\qquad A'_\mu(x')=A_\mu(x)\,,\qquad x'^\mu=x^\mu\,,
\tag{3.12}
$$

where Γ is a real parameter. Now we will argue that the action (3.10) is not generally invariant under the transformation (3.12). Firstly, using (3.12) we see that H_1 and H_2 transform as

$$
\begin{aligned}
H_1(Z') &= \frac{\lambda_1}{\Gamma^2|\mathbf{Z}|^2} = \Gamma^{-2} H_1(Z)\,, \\
H_2(Y') &= \frac{\lambda_2 \Gamma^2}{|\mathbf{Y}|^2} = \Gamma^2 H_2(Y)\,,
\end{aligned} \tag{3.13}
$$

so that $\sqrt{H_1 H_2}$ is invariant under (3.12). On the other hand using (3.13) we also see that Φ_1 and Φ_2 transform as

$$
\Phi_1(Z') = -\ln\Gamma + \Phi_1(Z)\,, \qquad \Phi_2(Y') = \ln\Gamma + \Phi_2(Y)\,. \tag{3.14}
$$

Then (3.2) implies that $\mathcal{H}_{pqr}(Z)$ and $\mathcal{H}_{\alpha\beta\gamma}(Y)$ transform as

$$
\mathcal{H}_{pqr}(Z') = \Gamma^{-1} H_{pqr}(Z)\,, \qquad \mathcal{H}_{\alpha\beta\gamma}(Y') = \Gamma H_{\alpha\beta\gamma}(Y)\,. \tag{3.15}
$$

Finally from the definition $\mathcal{H} = db$ we get that b'_{pq} and $b'_{\alpha\beta}$ are invariant under the transformations (3.12)

$$
b_{pq}(Z') = b_{pq}(Z)\,, \qquad b_{\beta\gamma}(Y') = b_{\beta\gamma}(Y)\,. \tag{3.16}
$$

This result implies that the expressions $b_{pq}\partial_\mu Z^p \partial_\nu Z^q\,, b_{\alpha\beta}\partial_\mu Y^\alpha \partial_\nu Y^\beta$ break the scale invariance of DBI action. In order to restore it we introduce two parameters m^1, m^2 as

$$
b_{pq}\partial_\mu Z^p \partial_\nu Z^q \rightarrow m^1 b_{pq}\partial_\mu Z^p \partial_\nu Z^q \tag{3.17}
$$

and also

$$
b_{\alpha\beta}\partial_\mu Y^\alpha \partial_\nu Y^\beta \rightarrow m^2 b_{\alpha\beta}\partial_\mu Y^\alpha \partial_\nu Y^\beta\,. \tag{3.18}
$$

Then using (3.16) we see that in order to have an action invariant under this scaling transformation parameters m_1 and m_2 have to transform as

$$
m'^1 = \Gamma^{-2} m^1\,, \qquad m'^2 = \Gamma^2 m^2\,. \tag{3.19}
$$

Finally using (3.12) and (3.19) we obtain following values of Φ^I and Ω^A

$$
\Phi_D^{Z^p} = Z^p\,, \qquad \Phi_D^{Y^\alpha} = -Y^\alpha\,, \qquad \Phi_D^{A_\mu} = 0\,, \qquad \Omega_D^{m^1} = -2m^1\,, \qquad \Omega_D^{m^2} = 2m^2\,. \tag{3.20}
$$

As a final remark note that the original action can be recovered from the action containing m_1 and m_2 simply by taking $m^1 = m^2 = 1$.

Now using (3.20) we get the form of the dilatation current

$$
\begin{aligned}
j_D^\mu &= -\frac{\delta\mathcal{L}}{\delta\partial_\mu Z^p}\Phi_D^{Z^p} - \frac{\delta\mathcal{L}}{\delta\partial_\mu Y^\alpha}\Phi_D^{Y^\alpha} \\
&= \tau_1 e^{-\Phi}\left(g_{pr} Z^p \partial_\nu Z^r \left(\mathbf{A}^{-1}\right)_S^{\nu\mu} + b_{pr} Z^p \partial_\nu Z^r \left(\mathbf{A}^{-1}\right)_A^{\nu\mu}\right)\sqrt{-\det\mathbf{A}} \,- \\
&\quad - \tau_1 e^{-\Phi}\left(g_{\alpha\beta} Y^\alpha \partial_\nu Y^\beta \left(\mathbf{A}^{-1}\right)_S^{\nu\mu} + b_{\alpha\beta} Y^\alpha \partial_\nu Y^\beta \left(\mathbf{A}^{-1}\right)_A^{\nu\mu}\right)\sqrt{-\det\mathbf{A}}\,.
\end{aligned} \tag{3.21}
$$

Finally, we will calculate the term that appears on the right hand side of the equation (2.24)

$$\begin{aligned}\frac{\delta\mathcal{L}}{\delta m^A}\Omega^A_D &= -2\frac{\delta\mathcal{L}}{\delta m^1}m^1 + 2\frac{\delta\mathcal{L}}{\delta m^2}m^2 \\ &= \tau_1 e^{-\Phi}\left(b_{pq}\partial_\mu Z^p\partial_\nu Z^q - b_{\alpha\beta}\partial_\mu Y^\alpha\partial_\nu Y^\beta\right)\left(\mathbf{A}^{-1}\right)^{\nu\mu}\sqrt{-\det\mathbf{A}}\,,\end{aligned} \tag{3.22}$$

where we have taken $m^1 = m^2 = 1$ in the end of calculation.

From the form of this anomaly term we see that the conservation of j^μ_D is restored for homogenous modes or for modes that are spatial depend only since then the anomaly term vanishes thanks to the antisymmetry of b. In this paper we restrict to the case of time dependent modes only. The case when world volume fields are spatial dependent will be discussed in separate publication.

Before we proceed we would like to write the expression for hamiltonian density of Dp-brane moving in nontrivial background. The reason why we do this is that the hamiltonian for D1-brane will play an important role in next section. More details considering the hamiltonian density for Dp-brane in general background can be found, for example in [37, 38]. In fact, using the analysis given there we have shown in [13] that the hamiltonian density for Dp-brane in curved background takes the form

$$\begin{aligned}\mathcal{H} &= \sqrt{-g_{00}}\sqrt{\mathcal{K}} + \pi^a b_{a0}\,, \\ \mathcal{K} &= \Pi_I g^{IJ}\Pi_J + \pi^a\gamma_{ab}\pi^b + d_a g^{ab} d_b + e^{-2\Phi}\tau_p^2 \det\mathbf{A}_{ab}\,, \\ \Pi_I &= P_I + \pi^a b_{aI} = P_I + \pi^a(b_{aI} + \partial_a X^J b_{JI})\,, \\ d_a &= \Pi_I\partial_a X^I + F_{ab}\pi^b\,, \\ \gamma_{ab} &= g_{ab} + \partial_a X^I g_{IJ}\partial_b X^J\,,\ \mathbf{A}_{ab} = g_{ab} + \partial_a X^I\partial_b X^J g_{IJ} + F_{ab}\,, \\ F_{ab} &= \partial_a A_b - \partial_b A_a + b_{ab} + b_{IJ}\partial_a X^I\partial_b X^J\,,\end{aligned} \tag{3.23}$$

where $a, b = 1,\dots p$ label the spatial coordinates on the world volume of Dp-brane. Note that in (3.23) we presume the background where the metric g and two form b are block diagonal so that $b_{aI} = g_{aI} = 0$. Moreover, we will also consider the background with $b_{a0} = 0$ and $g_{00} = -1$. Now using (3.23) it is easy to see that the canonical equations of motion for X^K and A_a take the form

$$\begin{aligned}\partial_0 X^I &= \frac{\delta H}{\delta P_I} = \frac{g^{IJ}\Pi_J + \partial_a X^I g^{ab} d_b}{\sqrt{\mathcal{K}}}\,, \\ \partial_0 A_a &= \frac{\delta H}{\delta \pi^a} = \frac{b_{aI} g^{IJ}\Pi_J + \gamma_{ab}\pi^b - F_{ab} g^{bc} d_c}{\sqrt{\mathcal{K}}}\,.\end{aligned} \tag{3.24}$$

On the other hand the equation of motion for P_I are much more complicated. However we will not need their explicit forms since we use an existence of conserved charges. On the hand the equation of motion for π^a will be useful

$$\partial_0\pi^a = -\frac{\delta H}{\delta A_a} = \partial_c\left[\frac{\pi^a g^{cd} d_d - \pi^c g^{ad} d_d}{\sqrt{\mathcal{K}}}\right] - \partial_b\left[\frac{e^{-2\Phi}\tau_p^2}{\sqrt{\mathcal{K}}}\left(\mathbf{A}^{-1}\right)^{ba}_A \det\mathbf{A}\right]. \tag{3.25}$$

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4. Time dependent world volume fields

In this section we will study the case when all world volume modes are time dependent only. Moreover, for the D1-brane the situation simplifies further since now $a, b = 1$ and hence the quantities defined in (3.23) are equal to

$$\gamma_{11} = g_{11} \,, \qquad \mathbf{A}_{11} = g_{11} \,, \qquad d_1 = 0 \,, \qquad \Pi_I = P_I \,. \tag{4.1}$$

Now thanks to the manifest rotation symmetry SO(4) in the subspaces spanned by coordinates $\mathbf{z} = (z^6, z^7, z^8, z^9)$ and $\mathbf{y} = (y^2, y^3, y^4, y^5)$ we can reduce the problem to the study of the motion on two dimensional subspaces, namely we will presume that only following world volume modes are excited

$$z^6 = R\cos\theta \,, \qquad z^7 = R\sin\theta \,, \tag{4.2}$$

and

$$y^2 = S\cos\psi \,, \qquad y^3 = S\sin\psi \,. \tag{4.3}$$

For these modes $\mathcal{K}$ defined in (3.23) is equal to

$$\begin{aligned}\mathcal{K} &= P_R g^{RR} P_R + P_S g^{SS} P_S + P_\theta g^{\theta\theta} P_\theta + P_\psi g^{\psi\psi} P_\psi + \pi^1 g_{11} \pi^1 + e^{-2\Phi} \tau_1^2 g_{11} \\ &= \frac{\tau_1^2 R^2 S^2}{g_1^2 g_2^2 \lambda_1 \lambda_2} \left(1 + \frac{\lambda_2 g_1^2 g_2^2}{\tau_1^2 S^2} P_R^2 + \frac{\lambda_1 g_1^2 g_2^2}{\tau_1^2 R^2} P_S^2 + \frac{\lambda_2 g_1^2 g_2^2}{\tau_1^2 R^2 S^2} P_\theta^2 + \frac{\lambda_1 g_1^2 g_2^2}{\tau_1^2 R^2 S^2} P_\theta^2 + \frac{g_1^2 g_2^2 \lambda_1 \lambda_2}{\tau_1^2 R^2 S^2} \pi^2\right) ,\end{aligned} \tag{4.4}$$

where on the second line we have taken the near horizon limit. Note that for simplicity of notation we define $\pi \equiv \pi^1 \,, A \equiv A_1$. Then using (4.4) the hamiltonian density (3.23) takes the form

$$\mathcal{H} = \frac{\tau_1 R S}{g_1 g_2 \sqrt{\lambda_1 \lambda_2}} \sqrt{1 + \frac{\lambda_2 g_1^2 g_2^2}{\tau_1^2 S^2} P_R^2 + \frac{\lambda_1 g_1^2 g_2^2}{\tau_1^2 R^2} P_S^2 + \frac{\lambda_2 g_1^2 g_2^2}{\tau_1^2 R^2 S^2} P_\theta^2 + \frac{\lambda_1 g_1^2 g_2^2}{\tau_1^2 R^2 S^2} P_\theta^2 + \frac{g_1^2 g_2^2 \lambda_1 \lambda_2}{\tau_1^2 R^2 S^2} \pi^2} \,. \tag{4.5}$$

In the same way we obtain that the zero component of the dilatation current j_D^0 (3.21) is equal to

$$j_D^0 \equiv d = -P_R R + P_S S \,. \tag{4.6}$$

Since the resulting densities do not depend on x^1 we will work with them instead of with corresponding charges.

Firstly, it is easy to see, using (4.5), that

$$\begin{aligned}\dot{P}_\theta &= -\frac{\delta\mathcal{H}}{\delta\theta} = 0 \,, \\ \dot{P}_\psi &= -\frac{\delta\mathcal{H}}{\delta\psi} = 0 \,, \\ \dot{\pi} &= -\frac{\delta\mathcal{H}}{\delta A} = 0\end{aligned} \tag{4.7}$$

so that P_ψ, P_θ and π are constant. On the other hand the equations of motion for R and S take the form

$$\begin{aligned}
\dot{R} &= \frac{\delta\mathcal{H}}{\delta P_R} = \frac{\tau_1 RS}{g_1 g_2\sqrt{\lambda_1\lambda_2}}\frac{\lambda_2 g_1^2 g_2^2}{\tau_1^2 S^2}\frac{P_R}{\sqrt{(\dots)}} = \frac{R^2 P_R}{\lambda_1\mathcal{H}}\,, \\
\dot{S} &= \frac{\delta\mathcal{H}}{\delta P_S} = \frac{\tau_1 RS}{g_1 g_2\sqrt{\lambda_1\lambda_2}}\frac{\lambda_1 g_1^2 g_2^2}{\tau_1^2 R^2}\frac{P_S}{\sqrt{(\dots)}} = \frac{S^2 P_S}{\lambda_2\mathcal{H}}\,,
\end{aligned} \tag{4.8}$$

where we have used the fact that $\mathcal{H}$ is conserved. Now in order to find the time dependence of R, S we proceed in the standard way and use an existence of the conserved dilatation density (4.6) and conserved hamiltonian density (4.5). We begin with simpler case when $P_\psi = P_\theta = \pi = 0$.

4.1 Case of zero P_θ, P_ψ, π

In this case the hamiltonian density is equal to

$$\mathcal{H} = \frac{\tau_1 RS}{g_1 g_2\sqrt{\lambda_1\lambda_2}}\sqrt{1 + \frac{\lambda_2 g_1^2 g_2^2}{\tau_1^2 S^2}P_R^2 + \frac{\lambda_1 g_1^2 g_2^2}{\tau_1^2 R^2}P_S^2}\,. \tag{4.9}$$

Let us also consider the situation when the dilatation density (4.6) vanishes and hence

$$\frac{P_R}{S} = \frac{P_S}{R}\,. \tag{4.10}$$

Inserting (4.8) into the equation (4.10) we get

$$\frac{\dot{R}}{R}\lambda_1 = \frac{\dot{S}}{S}\lambda_2 \tag{4.11}$$

that implies

$$R^{\frac{\lambda_1}{\lambda_2}}C = S\,, \tag{4.12}$$

where $C = e^{\frac{d_0}{\lambda_2}}$ is an integration constant and where the meaning of d_0 will be given below. With the help of (4.10) and (4.12) we can rewrite the hamiltonian density (4.9) into the form

$$\mathcal{H} = \frac{CR^{(\lambda_1+\lambda_2)/\lambda_2}}{g_1 g_2\sqrt{\lambda_1\lambda_2}}\sqrt{\tau_1^2 + g_1^2 g_2^2(\lambda_1+\lambda_2)C^{-2}P_R^2 R^{-2\lambda_1/\lambda_2}} \tag{4.13}$$

that allows us to express P_R as a function of $\mathcal{H}$ and R

$$P_R^2 = \frac{\mathcal{H}^2\lambda_1\lambda_2}{(\lambda_1+\lambda_2)R^2} - \frac{C^2\tau_1^2}{g_1^2 g_2^2(\lambda_1+\lambda_2)}R^{2\lambda_1/\lambda_2}\,. \tag{4.14}$$

Finally, from (4.8) and (4.14) we get a differential equation for R in the form

$$\dot{R} = \pm R\sqrt{\frac{\lambda_2}{\lambda_1(\lambda_1+\lambda_2)} - \frac{\tau_1^2 R^{\frac{2(\lambda_1+\lambda_2)}{\lambda_2}}}{C^2(g_1g_2)^2\lambda_1^2\mathcal{H}^2(\lambda_1+\lambda_2)}} \tag{4.15}$$

that has the solution

$$R = \left(\frac{\lambda_1\lambda_2(g_1g_2)^2\mathcal{H}^2}{C^2\tau_1^2}\right)^{\frac{\lambda_2}{2(\lambda_1+\lambda_2)}} \left(\frac{1}{\cosh\sqrt{\frac{\lambda_1+\lambda_2}{\lambda_1\lambda_2}}t}\right)^{\frac{\lambda_2}{\lambda_1+\lambda_2}} , \tag{4.16}$$

where we have chosen the initial condition that for $t = 0$ D1-brane is in its turning point where $\dot{R} = 0$. Finally, using (4.12) and (4.16) we obtain the time dependence of S

$$S = C\left(\frac{\lambda_1\lambda_2(g_1g_2)^2\mathcal{H}^2}{C^2\tau_1^2}\right)^{\frac{\lambda_1}{2(\lambda_1+\lambda_2)}} \left(\frac{1}{\cosh\sqrt{\frac{\lambda_1+\lambda_2}{\lambda_1\lambda_2}}t}\right)^{\frac{\lambda_1}{\lambda_1+\lambda_2}} . \tag{4.17}$$

The physical picture of the time evolution of D1-brane is following. The D1-brane leaves the world volume of I-brane at $t = -\infty$, reaches its turning point at $t = 0$ and then it again moves to the world volume of I-brane. It would be certainly very interesting to propose the holographic interpretation of this dynamic situation as was done in case of Dp-brane in NS5-brane background in [29].

Let us now consider more general case when the dilatation current j_D^0 is nonzero and hence

$$d = -P_R R + P_S S = -\lambda_1\mathcal{H}\frac{\dot{R}}{R} + \lambda_2\mathcal{H}\frac{\dot{S}}{S} , \tag{4.18}$$

where we have used the equations of motion (4.8). The integration of the equation (4.18) gives

$$\frac{d}{\mathcal{H}}t + d_0 = \ln\left(\frac{S^{\lambda_2}}{R^{\lambda_1}}\right) \Rightarrow S = R^{\lambda_1/\lambda_2}e^{\frac{1}{\lambda_2}(\frac{d}{\mathcal{H}}t+d_0)} . \tag{4.19}$$

Then using (4.18) and (4.19) we can express the hamiltonian density (4.9) as

$$\begin{aligned}\mathcal{H} &= \sqrt{\frac{R^2S^2\tau_1^2}{(g_1g_2)^2(\lambda_1\lambda_2)} + \frac{1}{\lambda_1}P_R R^2 + \frac{1}{\lambda_2}P_S S} \\ &= \sqrt{\frac{R^2S^2\tau_1^2}{(g_1g_2)^2(\lambda_1\lambda_2)} + \frac{\lambda_1+\lambda_2}{\lambda_1\lambda_2}R^2P_R^2 + \frac{d^2}{\lambda_2} + 2P_R R\frac{d}{\lambda_2}} .\end{aligned} \tag{4.20}$$

From (4.20) we now express P_R as function of R, S

$$P_R = -\frac{d\lambda_1}{(\lambda_1+\lambda_2)R} \pm \frac{\lambda_1\lambda_2}{(\lambda_1+\lambda_2)R}\sqrt{\frac{\mathcal{H}^2(\lambda_1+\lambda_2)}{\lambda_1\lambda_2} - \frac{d^2}{\lambda_1\lambda_2} - \frac{R^2S^2\tau_1^2}{\lambda(g_1g_2)^2\lambda_1\lambda_2}} . \tag{4.21}$$

Consequently the equation of motion for R takes the form

$$\dot{R} = \frac{R^2P_R}{\lambda_1\mathcal{H}} = -d\frac{1}{(\lambda_1+\lambda_2)}R \pm \frac{\lambda_2 R}{(\lambda_1+\lambda_2)\mathcal{H}}\sqrt{\frac{\mathcal{H}^2(\lambda_1+\lambda_2)}{\lambda_1\lambda_2} - \frac{d^2}{\lambda_1\lambda_2} - \frac{(\lambda_1+\lambda_2)R^2S^2\tau_1^2}{(g_1g_2)^2(\lambda_1\lambda_2)^2}} . \tag{4.22}$$

In order to solve the equation (4.22) we consider following ansatz

$$R = K(t)e^{-\frac{d}{(\lambda_1+\lambda_2)\mathcal{H}}t} \tag{4.23}$$

and insert it to the equation (4.22). Then we obtain a differential equation for K

$$\dot{K} = \pm\frac{K}{\sqrt{\lambda_1+\lambda_2}}\sqrt{\frac{\lambda_2}{\lambda_1} - \frac{d^2}{\mathcal{H}^2(\lambda_1+\lambda_2)}\frac{\lambda_2}{\lambda_1} - \frac{\tau_1^2 e^{2d_0/\lambda_2}}{\mathcal{H}^2(g_1g_2)^2\lambda_1^2}K^{\frac{2(\lambda_1+\lambda_2)}{\lambda_2}}}\,, \tag{4.24}$$

where we have used the fact that for (4.23) we have

$$R^2S^2 = R^{\frac{2(\lambda_1+\lambda_2)}{\lambda_2}}e^{\frac{2}{\lambda_2}(\frac{d}{\mathcal{H}}t+d_0)} = K^{\frac{2(\lambda_1+\lambda_2)}{\lambda_2}}e^{2d_0/\lambda_2}\,. \tag{4.25}$$

The equation (4.24) can be straightforwardly integrated with the result

$$\begin{aligned} K =& \left(\frac{\sqrt{\lambda_1\lambda_2}\sqrt{1-\frac{d^2}{\mathcal{H}^2(\lambda_1+\lambda_2)}}g_1g_2\mathcal{H}e^{-\frac{d_0}{\lambda_2}}}{\tau_1}\right)^{\lambda_2/(\lambda_1+\lambda_2)}\times \\ &\times\left(\frac{1}{\cosh\sqrt{\frac{\lambda_1+\lambda_2}{\lambda_1\lambda_2}}\sqrt{1-\frac{d^2}{\mathcal{H}^2(\lambda_1+\lambda_2)}}t}\right)^{\lambda_2/(\lambda_1+\lambda_2)}\,. \end{aligned} \tag{4.26}$$

Finally using the result given above and (4.23) we obtain the time dependence of R

$$\begin{aligned} R =& e^{-\frac{1}{\sqrt{\lambda_1+\lambda_2}}\left(\frac{1}{\sqrt{\lambda_1+\lambda_2}}\frac{d}{\mathcal{H}}t+\frac{d_0}{\sqrt{\lambda_1+\lambda_2}}\right)}\left(\frac{\sqrt{\lambda_1\lambda_2}\sqrt{1-\frac{d^2}{\mathcal{H}^2(\lambda_1+\lambda_2)}}g_1g_2\mathcal{H}}{\tau_1}\right)^{\frac{\lambda_2}{(\lambda_1+\lambda_2)}}\times \\ &\times\left(\frac{1}{\cosh\sqrt{\frac{\lambda_1+\lambda_2}{\lambda_1\lambda_2}}\sqrt{1-\frac{d^2}{\mathcal{H}^2(\lambda_1+\lambda_2)}}t}\right)^{\frac{\lambda_2}{(\lambda_1+\lambda_2)}}\,. \end{aligned} \tag{4.27}$$

The time dependence of S then follows from (4.19) and from (4.27).

An importance of the solution given above will be seen below when we compare it with the solution obtained in the background introduced in [1]. To see this let us again write the lagrangian for time dependent D1-brane in the near horizon limit

$$\mathcal{L} = -\frac{\tau_1}{g_1g_2}\sqrt{\frac{R}{\lambda_1}}\sqrt{\frac{S}{\lambda_2}}\sqrt{1-\frac{\lambda_1}{R^2}\dot{R}^2-\frac{\lambda_2}{S^2}\dot{S}^2}\,. \tag{4.28}$$

As the first step let us introduce two modes ϕ_1 and ϕ_2 defined as

$$\sqrt{\lambda_1}\frac{dR}{R} = d\phi_1\,, \qquad \sqrt{\lambda_2}\frac{dS}{S} = d\phi_2 \tag{4.29}$$

or equivalently

$$R = e^{\frac{\phi_1}{\sqrt{\lambda_1}}}\,, \qquad S = e^{\frac{\phi_2}{\sqrt{\lambda_2}}}\,. \tag{4.30}$$

If we insert (4.30) into (4.28) we obtain

$$\mathcal{L} = -\frac{\tau_1 e^{(\frac{\phi_1}{\sqrt{\lambda_1}}+\frac{\phi_2}{\sqrt{\lambda_2}})}}{\sqrt{\lambda_1\lambda_2}g_1g_2}\sqrt{1-\dot{\phi}_1^2-\dot{\phi}_2^2}\,. \tag{4.31}$$

Following [1] we introduce two modes ϕ, x^2 through the relation

$$Q\phi = \frac{1}{\sqrt{\lambda_1}}\phi_1 + \frac{1}{\sqrt{\lambda_2}}\phi_2\,, \qquad Qx^2 = \frac{1}{\sqrt{\lambda_2}}\phi_1 - \frac{1}{\sqrt{\lambda_1}}\phi_2\,, \tag{4.32}$$

where

$$Q = \frac{1}{\sqrt{\lambda}}\,, \qquad \frac{1}{\lambda} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}\,. \tag{4.33}$$

Note that the inverse transformations to (4.32) take the forms

$$\begin{aligned} \phi_1 &= \frac{1}{\sqrt{\lambda_1+\lambda_2}}\left(\sqrt{\lambda_1}x^2 + \sqrt{\lambda_2}\phi\right)\,, \\ \phi_2 &= \frac{1}{\sqrt{\lambda_1+\lambda_2}}\left(\sqrt{\lambda_1}\phi - \sqrt{\lambda_2}x^2\right)\,. \end{aligned} \tag{4.34}$$

Then using (4.34) we can express the lagrangian (4.31) as

$$\mathcal{L} = -\frac{\tau_1}{\sqrt{\lambda_1\lambda_2}g_1g_2}e^{Q\phi}\sqrt{1-(\partial_0x^2)^2-(\partial_0\phi)^2}\,. \tag{4.35}$$

This result demonstrates an enhancement of symmetry in the near horizon region as was previously shown in [1]. In fact the form of the lagrangian (4.35) suggests to interpret x^2 as a mode that describes the location of D1-brane in additional world volume direction of $2+1$ dimensional object. Moreover, due to the fact that the lagrangian (4.35) does not explicitly depend on x^2 the momentum P_2 conjugate to x^2

$$P_2 = \frac{\delta\mathcal{L}}{\delta\partial_0x^2} = \frac{\tau_1 e^{Q\phi}}{\sqrt{\lambda_1\lambda_2}g_1g_2}\frac{\partial_0x^2}{\sqrt{1-(\partial_0x^2)^2-(\partial_0\phi)^2}} \tag{4.36}$$

is conserved. We also see that the hamiltonian H defined as

$$\begin{aligned} H &= \partial_0x^2P_2 + \partial_0\phi P_\phi - \mathcal{L} = \frac{\tau_1e^{Q\phi}}{g_1g_2\sqrt{\lambda_1\lambda_2}}\frac{1}{\sqrt{1-(\partial_0x^2)^2-(\partial_0\phi)^2}} \\ &= \sqrt{P_2^2+P_\phi^2+\frac{\tau_1^2}{\lambda_1\lambda_2(g_1g_2)^2}e^{2Q\phi}} \end{aligned} \tag{4.37}$$

is also conserved and equal to the energy E. Now using (4.37) we express P_ϕ as function of ϕ so that the equation of motion $\dot{\phi} = \frac{P_\phi}{H}$ implies a differential equation for ϕ

$$\frac{d\phi}{\sqrt{1-\frac{\tau_1^2}{(E^2-P_2^2)(g_1g_2)^2\lambda_1\lambda_2}e^{2Q\phi}}} = \pm\sqrt{1-\frac{P_2^2}{E^2}}dt\,. \tag{4.38}$$

that has the solution

$$e^{Q\phi} = \frac{\sqrt{E^2-P_2^2}(g_1g_2)\sqrt{\lambda_1\lambda_2}}{\tau_1}\frac{1}{\cosh Q\sqrt{1-\frac{P_2^2}{E^2}}t} \tag{4.39}$$

or equivalently

$$\phi = \ln\left(\frac{\sqrt{E^2-P_2^2}(g_1g_2)\sqrt{\lambda_1\lambda_2}}{\tau_1}\right)^{\sqrt{\frac{\lambda_1\lambda_2}{\lambda_1+\lambda_2}}} + \ln\left(\frac{1}{\cosh Q\sqrt{1-\frac{P_2^2}{E^2}}t}\right)^{\sqrt{\frac{\lambda_1\lambda_2}{\lambda_1+\lambda_2}}} . \quad (4.40)$$

Note that we have chosen the initial condition corresponding to D1-brane that reaches its turning point at $t=0$. Then from the inverse transformations (4.34) and the fact that $x^2 = \frac{P_2}{E}t + x_0^2$ we obtain the time dependence of ϕ_1 and ϕ_2

$$\begin{aligned}\phi_1 &= \sqrt{\frac{\lambda_1}{\lambda_1+\lambda_2}}\left(\frac{P_2}{E}t + x_0^2\right) + \sqrt{\frac{\lambda_2}{\lambda_1+\lambda_2}}\phi\,, \\ \phi_2 &= -\sqrt{\frac{\lambda_2}{\lambda_1+\lambda_2}}\left(\frac{P_2}{E}t + x_0^2\right) + \sqrt{\frac{\lambda_1}{\lambda_1+\lambda_2}}\phi\,. \end{aligned} \quad (4.41)$$

Using (4.30) and also (4.40) we finally obtain

$$\begin{aligned} R &= \exp\left(\sqrt{\frac{1}{\lambda_1+\lambda_2}}\left(\frac{P_2}{E}t+x_0^2\right)\right)\left(\frac{\sqrt{E^2-P_2^2}(g_1g_2)^2\lambda_1\lambda_2}{\tau_1^2}\right)^{\frac{\lambda_2}{2(\lambda_1+\lambda_2)}} \times \\ &\times \left(\frac{1}{\cosh Q\sqrt{1-\frac{P_2^2}{E^2}}t}\right)^{\frac{\lambda_2}{\lambda_1+\lambda_2}}, \\ S &= \exp\left(-\sqrt{\frac{1}{\lambda_1+\lambda_2}}\left(\frac{P_2}{E}t+x_0^2\right)\right)\left(\frac{\sqrt{E^2-P_2^2}(g_1g_2)^2\lambda_1\lambda_2}{\tau_1^2}\right)^{\frac{\lambda_1}{2(\lambda_1+\lambda_2)}} \times \\ &\times \left(\frac{1}{\cosh Q\sqrt{1-\frac{P_2^2}{E^2}}t}\right)^{\frac{\lambda_1}{\lambda_1+\lambda_2}} . \end{aligned} \quad (4.42)$$

If we compare this solution with (4.27) we get following identification

$$P_2 = \frac{d}{\sqrt{\lambda_1+\lambda_2}}\,, \qquad x_0^2 = \frac{d_0}{\sqrt{\lambda_1+\lambda_2}}\,. \quad (4.43)$$

In summary, we have shown that the scaling symmetry that was found for D1-brane moving in the original background has clear relation to the enhancement symmetry presented in [1]. This result also gives a physical interpretation of the dilatation charge d that after rescaling corresponds to the conserved momentum P_2 in the new flat direction.

4.2 Case of nonzero P_ψ, P_θ and π

For nonzero P_ψ, P_θ and π the hamiltonian density takes the form

$$\mathcal{H} = \frac{RS}{g_1g_2\sqrt{\lambda_1\lambda_2}}\sqrt{\tau_1^2 + \frac{\lambda_2 g_1^2 g_2^2}{S^2}P_R^2 + \frac{\lambda_1 g_1^2 g_2^2}{R^2}P_S^2 + \frac{g_1^2g_2^2}{R^2S^2}G}\,, \quad (4.44)$$

where we have defined

$$G = \lambda_2 P_\theta^2 + \lambda_1 P_\psi^2 + \lambda_1 \lambda_2 \pi^2 \,. \tag{4.45}$$

As usual, the equations of motion for R and S take the form

$$\begin{aligned}\dot{R} &= \frac{R^2 P_R}{\lambda_1 \mathcal{H}} \,, \\ \dot{S} &= \frac{S^2 P_S}{\lambda_2 \mathcal{H}} \,. \end{aligned} \tag{4.46}$$

For simplicity we restrict ourselves to the case of the vanishing dilatation density $d = 0$ bear in mind that this analysis can be easily extended to the case of nonzero d as we have shown in the previous section.

For $d = 0$ we again obtain the relation between R and S in the form

$$R^{\frac{\lambda_1}{\lambda_2}} C = S \,, C = e^{\frac{d_0}{\lambda_2}} \,. \tag{4.47}$$

Using also the fact that $P_S = \frac{R}{S} P_R$ we express $\mathcal{H}$ as function of R and P_R only

$$\begin{aligned}\mathcal{H} &= \frac{RS}{g_1 g_2 \sqrt{\lambda_1 \lambda_2}} \sqrt{\tau_1^2 + (g_1 g_2)^2 \left(\lambda_2 + \lambda_1\right) \frac{P_R^2}{S^2} + \frac{g_1^2 g_2^2}{R^2 S^2} G} \\ &= \frac{C R^{\frac{\lambda_1 + \lambda_2}{\lambda_2}}}{g_1 g_2 \sqrt{\lambda_1 \lambda_2}} \sqrt{\tau_1^2 + (g_1 g_2)^2 (\lambda_1 + \lambda_2) C^{-2} P_R^2 R^{-2\lambda_1/\lambda_2} + (g_1 g_2)^2 G C^{-2} R^{-2(\lambda_1+\lambda_2)/\lambda_2}} \end{aligned} \tag{4.48}$$

If we express P_R from (4.48) and insert it to the first equation in (4.46) we obtain the differential equation for R

$$\dot{R} = \pm \left(\frac{1}{\lambda_1^2 \mathcal{H}^2} \left[\frac{\mathcal{H}^2 \lambda_1 \lambda_2}{(\lambda_1 + \lambda_2)} - \frac{G}{(\lambda_1 + \lambda_2)} \right] R^2 - \frac{1}{\lambda_1^2 \mathcal{H}^2} \frac{C^2 \tau_1^2}{(g_1 g_2)^2 (\lambda_1 + \lambda_2)} R^{4 + 2\lambda_1/\lambda_2} \right)^{1/2} \tag{4.49}$$

that has the solution

$$\begin{aligned} R &= \left(\frac{\lambda_1 \lambda_2 (g_1 g_2)^2 \mathcal{H}^2}{C^2 \tau_1^2} - \frac{G (g_1 g_2)^2}{C^2 \tau_1^2} \right)^{\frac{\lambda_2}{2(\lambda_1 + \lambda_2)}} \left(\frac{1}{\cosh \sqrt{\frac{\lambda_1 + \lambda_2}{\lambda_1 \lambda_2} - \frac{G(\lambda_1 + \lambda_2)}{\mathcal{H}^2 (\lambda_1 \lambda_2)^2}}\, t} \right)^{\frac{\lambda_2}{\lambda_1 + \lambda_2}} \,, \\ S &= C \left(\frac{\lambda_1 \lambda_2 (g_1 g_2)^2 \mathcal{H}^2}{C^2 \tau_1^2} - \frac{G (g_1 g_2)^2}{C^2 \tau_1^2} \right)^{\frac{\lambda_1}{2(\lambda_1 + \lambda_2)}} \left(\frac{1}{\cosh \sqrt{\frac{\lambda_1 + \lambda_2}{\lambda_1 \lambda_2} - \frac{G(\lambda_1 + \lambda_2)}{\mathcal{H}^2 (\lambda_1 \lambda_2)^2}}\, t} \right)^{\frac{\lambda_1}{\lambda_1 + \lambda_2}} \,, \end{aligned} \tag{4.50}$$

where on the second line we have used the relation (4.47). We see that nonzero values of P_θ, P_ψ and π do not significantly change the resulting dynamics. The same situation was previously reported in the papers devoted to the study of the dynamics of Dp-brane in NS5-brane background.

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